$$\int_0^{2\pi} \cos^m \theta \, d\theta =$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\cos^m \theta = \left(\frac{1}{2}\right)^m \left(e^{i\theta} + e^{-i\theta}\right)^m$$

$$= \left(\frac{1}{2}\right)^m \sum_{k=0}^{m} \binom{m}{k} \left(e^{i\theta}\right)^k \left(e^{-i\theta}\right)^{m-k}$$

$$= \left(\frac{1}{2}\right)^m \sum_{k=0}^{m} \binom{m}{k} e^{k i\theta + (m-k)(-i\theta)}$$

$$= \left(\frac{1}{2}\right)^m \sum_{k=0}^{m} \binom{m}{k} e^{(-m+2k) i\theta}$$

$$z = e^{i\theta}$$

$$dz = i e^{i\theta} d\theta$$

$$d\theta = \frac{dz}{iz}$$

$$\int_0^{2\pi} \cos^m \theta \, d\theta = \oint_{|z|=1} \left(\frac{1}{2}\right)^m \sum_{k=0}^{m} \binom{m}{k} z^{(2k-m)} \frac{dz}{iz}$$

$$\int_0^{2\pi} \cos^m\theta \, d\theta = \left(\frac{1}{2}\right)^m \oint_{|z|=1} \sum \quad - \quad -$$

$$= -i\left(\frac{1}{2}\right)^m \sum_{k=0}^{m} \binom{m}{k} \underbrace{\oint z^{2k-m-1} \, dz}$$

האינטגרל הזה שווה ל- 0

אלא אם כן

$$2k-m-1=-1$$

$$2k-m=0$$

$$k=\frac{m}{2}$$

במקרה זה, האינטגרל

שווה ל- $2\pi i$

$$\oint_{|z|=1} z^n \, dz = \begin{cases} 0, & n \neq -1 \\ 2\pi i, & n=-1 \end{cases}$$

$$\int_0^{2\pi} \cos^m\theta \, d\theta = -i\left(\frac{1}{2}\right)^m \binom{m}{\frac{m}{2}} 2\pi i$$

$$= 2^{-m+1} \binom{m}{\frac{m}{2}} \pi .$$